Torque

Torque is the rotational equivalent of force. It takes into account not only how large and in what direction a force acts, but also where it is applied to an object.

$$\boxed{\boldsymbol{\tau} = \mathbf{d} \times \mathbf{F} = dF \sin\theta}$$

Where **d** is the vector describing the location of the force as measured from the axis of rotation. This is also called the "moment arm". **F** is the force applied and θ is the angle between the moment arm and the force. The direction of torque is along the axis of rotation and is positive or negative according to whether it tends to rotate the object counter–clockwise or clockwise.

Rotation is measured as an angle expressed in radians. Radians are used for mathematical convenience. Each angular quantity is related to a linear quantity of a particle on the object that is rotating.

$\theta =$ rotational angle in radians.

$s =$ arc length in meters $= R\theta$

$\omega =$ angular velocity in radians per second.

$v =$ linear velocity of a point on the object $= R\omega$

$\alpha =$ angular acceleration in radians per second squared.

$a =$ linear acceleration of a point on the object $= R\alpha$

Each of the kinematics and dynamics equations for linear motion has a corresponding rotational equation that differs only in the variables that are used.

$$\mathbf{d} = \mathbf{d}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2$$

$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + \boldsymbol{\omega}_0 t + \tfrac{1}{2}\boldsymbol{\alpha}t^2$$

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}$$

$$\Sigma\mathbf{F} = m\mathbf{a}$$

$$\Sigma\boldsymbol{\tau} = I\boldsymbol{\alpha}$$

I is the rotational inertia or "moment of inertia" of an object. The difficulty in changing the rotation of an object is dependent not only on its mass but also on how the mass is distributed. The rotational inertia takes this into account.

For an object to be in complete equilibrium, it must be in both linear and rotational equilibrium. That is, both the forces acting on it and the torques acting on it must have a net of zero. When an object is not moving and in equilibrium, it is permissible to choose any point as the point of rotation since torques must add to zero everywhere. Choosing a good point of rotation will make the solution of a problem more simple.

$$\Sigma\boldsymbol{\tau} = 0$$

$$\Sigma\mathbf{F}_x = 0$$

$$\Sigma\mathbf{F}_y = 0$$

When solving problems with beams and benches and boards, it is assumed that the center of mass of the object is at the geometric center. If it is not, the problem must state that fact. The entire mass of an object can be assumed to be at the center of mass when solving problems and the force of gravity acts there.